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Preferential deformation in a mild steel specimen under fast rate compressive loading

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Specimens of mild steel were subjected to fast-rate uniaxial compressive loading at liquid nitrogen temperatures such that overall plastic deformation was slight. These conditions should also represent the initial phase of more severe deformation treatments. Deformation seemed to have occurred completely by twinning. By studying and analysing the inclinations of the twin traces with the loading direction as observed on sections of the specimens containing the applied force direction it was found that twinning had occurred preferentially along certain directions relative to the applied force direction. A statistical model based on a minimum threshold value of 0.25 of the Schmid factor for twinning and a suppression factor for twinning in directions opposing the applied force was adopted and found to be successful in accounting for the observed experimental frequency distribution of different inclinations of the twin planes to the applied force direction.

1. Introduction

Under a sufficiently great applied force, a polycrystalline metal will plastically deform by slip or twinning, or both, so that the metal may yield appropriately to form the shape promoted by the applied forces. If the applied forces are strongly directional one would expect that certain slip or twinning planes would be more active than others, depending on their orientation relative to the applied forces, and on the plastic flow required to achieve the resultant shape. The situation may be complicated by the fact that grain accommodation processes may also operate.

The present paper reports on a study on the orientation of planes relative to the applied forces which favour or disfavour the initial plastic deformation process in a simple situation – that of a specimen of simple shape, subjected to a uniaxial compressive force tending to flatten it. The specimen is a solid right-angled parallelepiped of mild steel, of dimensions $13.5 \times 13.5 \times 10.0\ mm^3$, cooled down to liquid nitrogen temperatures, and stood upright on a flat rigid base with its square faces vertical, and onto whose horizontal top face is dropped a 1.9 kg weight from various heights, thus producing fast-rate compressive loading in the vertical direction. Mild steel and fast-rate loading at very low temperatures have been selected because, under these circumstances, ferrite, the dominant phase in the steel, deforms almost exclusively by twinning. Such twinning occurs along $\{1\,1\,2\}$ with shear along $\langle 1\,1\,1 \rangle$ [1], thus there are twelve 'twinning systems' in any ferrite grain. Although not all of these will be operative under a given loading condition because of the polar nature of deformation twinning there should be a sufficient number which can be activated to ensure that deformation along preferred directions relative to the applied forces will not be hindered by an insufficiency to twin along. The deformation twins in the ferrite also give clear metallographic traces (Fig. 1) which permit information to be gathered as to which orientations of $\{1\,1\,2\}$ planes promote the deformation of the specimen. In essence, the angles ψ between twin trace directions and the impact direction on a vertical metallographic section of the specimen are measured for many twin observations. If, in the deformation of the specimen, there is no preferred orientation of $\{1\,1\,2\}$ planes for twinning, then there will be found an equal frequency of occurrence of ψ angles of any value. On the other hand, if there are preferred orientations of $\{1\,1\,2\}$ planes for twinning, then some values of ψ will occur more frequently than others. If there is axial symmetry about the impact direction then it will also be possible to deduce the frequency distribution of the actual angles of inclination θ of occurring twins relative to the impact direction from the ψ frequency distribution by a method such as that of Scriven and Williams [2].

2. Experimental procedure

Ten parallelepiped specimens were machined from an original 16 mm square bar of JIS 3123 mild steel having the nominal composition: 0.1% C, 0.65% Mn, 0.2% Si, 0.02% S, and 0.02% P. Eight of these specimens (to be referred to as type A) had been machined

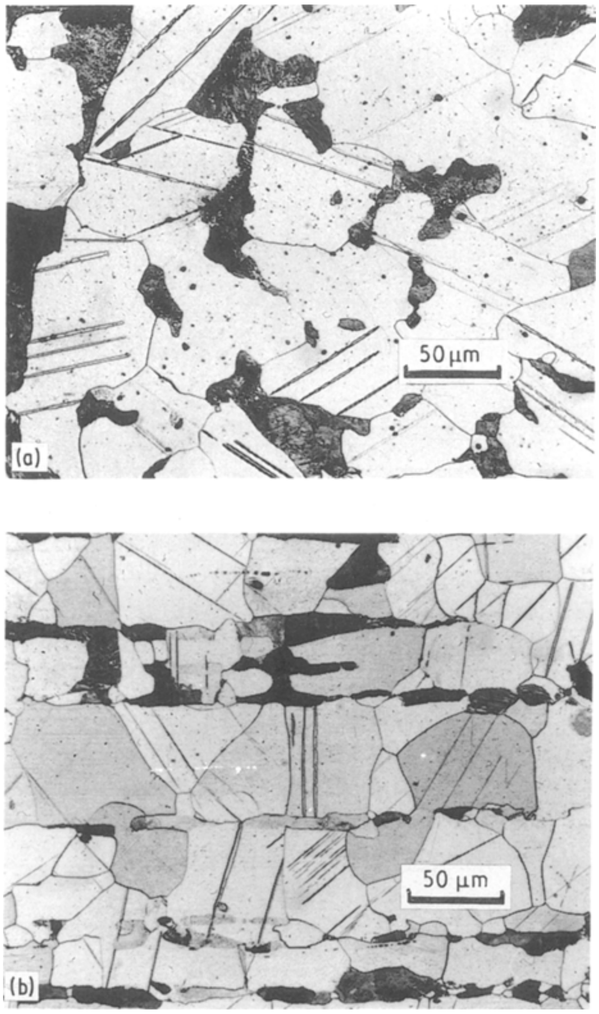

Figure 1 Typical microstructure in (a) type A specimens and (b) type B specimens showing ferrite and pearlite and the deformation twins in the ferrite. Sections made along a plane containing the impact axis.

such that the $13.5 \times 13.5\ mm^2$ square faces were perpendicular to the rolling direction of the bar; the remaining two specimens (to be called type B) were machined with their square faces and an opposite pair of sides of these faces parallel to the rolling direction of the bar. The bar had been initially austenitized at 1100 °C for 20 min, and then cooled very slowly, so as to produce relatively large grains of ferrite which have been found to be more amenable to deformation twinning [3].

The fast-rate loading of the specimens was done as follows. The specimens, prior to deformation, were first immersed in liquid nitrogen, and then quickly transferred to a massive steel base plate lying flat on a concrete floor. They were placed standing on the plate with their square faces vertical and with the rolling axis vertical in the case of type B specimens. The base plate itself had been pre-cooled to the liquid nitrogen temperature, so that the specimens would not acquire heat too rapidly from the surroundings. The 1.9 kg weight used for the impact was a solid steel cylinder 130 mm long and 50 mm in diameter. It was first raised to the required height above the specimen with its length vertical, and, when the specimen had been positioned on the base plate, dropped immediately through a vertical PVC pipe which helped guide it to fall squarely on the specimen. The impact blow was thus delivered perpendicular to the rolling direction in the case of type A specimens, and parallel to the rolling direction in the case of type B specimens. In this way one specimen of type A was subjected to a drop height of 3 m, three to a drop height of 4 m, and four to a drop height of 5 m. In the case of the specimens of type B one was subjected to a drop height of 4 m and the other 5 m.

Three different drop heights were used to allow a study of behaviour under progressive degrees of severity of impact. The particular drop heights used were suggested from earlier trials which showed that they produced plastic deformations which were not so severe as to distort twin shapes which would make measurements of the angles ψ difficult. Further, the intention was to have only slight plastic strain to substantially reduce or avoid grain accommodation effects which would make theoretical analysis of the deformation behaviour difficult. In fact, on the macroscopic scale, hardly any permanent deformation could be noted. Later additional tests on further specimens showed that the permanent compressive strain sustained in the specimens (as measured from displaced gauge marks on the specimens) averaged 0.6% and 1.7% for drop heights of 4 and 5 m, respectively. By means of a thermocouple junction attached to the specimens, their temperatures at time of impact could be ascertained, and these varied between −178 and −190 °C for the different specimens. This variation will be seen later to be not critical in relation to the findings of this work.

The plastic wave propagation in specimens subjected to high strain rates has been discussed by Lee and Wolf [4]. It may be deduced from their considerations that in the present work where small specimens are being impacted by a heavy massive weight at not too high velocities ($\cong 10\ m\,s^{-1}$) the maximum stress developing at any point in the specimens will be the same as at some other point.

After impact, the specimens were metallographically prepared and examined on a section which was parallel to the $13.5 \times 13.5\ mm^2$ faces. Systematically scanning the specimens, the angles ψ between the impact direction and the traces of encountered twin formations were measured with the aid of an eyepiece goniometer. A twin formation, in the present context, refers to all the twins in a grain which are aligned in the same direction. Measurements were taken of more than 1000 twin formations in more than 800 grains for each specimen. From these measurements the number of occurrences of angles ψ in 5° intervals were ascertained and the frequencies tabulated in Table I. It is noted that some ψ values occur more frequently than others suggesting that twins form more favourably along (1 1 2) planes oriented along certain directions with respect to the impact direction than others. It was observed that for about 70% of the grains with twins twinning had occurred only along one direction in these grains. In about 25% of grains with twins the twins formed along two directions. In only about 5%

TABLE I Percentage frequency of occurrence of ψ

ψ (deg)	Type A 3 m drop								Type B 4 m drop	Type B 5 m drop
	3 m drop	4 m drop			5 m drop				4 m drop	5 m drop
	Case 1	Case 1	Case 2	Case 3	Case 1	Case 2	Case 3	Case 4	Case 1	Case 1
0–5	0.25	0.39	0.00	0.00	0.14	0.00	0.00	0.00	0.18	0.00
5–10	0.34	0.32	0.00	0.00	0.21	0.09	0.00	0.08	0.00	0.00
10–15	0.42	0.55	0.64	0.65	0.83	0.28	0.41	0.24	0.27	0.08
15–20	0.76	1.58	1.19	1.02	1.24	0.46	0.91	0.49	0.82	0.66
20–25	1.53	2.05	1.43	1.21	2.97	0.83	1.24	1.94	1.64	1.82
25–30	2.37	3.95	3.82	2.32	3.04	2.50	3.07	3.81	3.19	2.98
30–35	5.17	4.50	6.12	3.25	4.84	4.55	4.39	5.43	3.64	5.63
35–40	6.45	7.73	6.60	6.22	5.05	5.38	4.64	4.78	10.38	7.53
40–45	7.72	7.26	7.55	7.06	8.64	5.01	5.06	8.67	10.66	7.12
45–50	7.80	7.89	7.87	8.64	7.54	6.22	9.78	11.43	10.93	8.11
50–55	7.72	8.05	9.30	11.61	9.06	9.55	10.03	9.81	12.20	8.44
55–60	10.60	9.23	12.08	11.61	8.30	9.28	10.61	9.40	9.20	10.18
60–65	10.18	9.39	10.33	9.29	8.92	8.72	9.12	8.83	7.29	9.35
65–70	7.38	7.34	8.35	6.78	8.37	8.35	9.20	10.29	7.29	8.94
70–75	8.57	8.13	8.43	9.47	8.57	11.22	8.62	7.29	6.92	8.61
75–80	8.23	8.76	5.17	9.10	8.16	11.87	9.04	6.56	7.19	8.53
80–85	6.96	6.39	6.20	6.78	7.95	8.53	8.54	6.40	4.64	6.95
85–90	7.55	6.47	4.93	5.01	6.22	7.14	5.31	4.53	3.55	5.05
Total occurrence	1179	1267	1258	1077	1446	1078	1206	1234	1098	1208

of the grains did twins occur along more than two directions.

Fig. 1a and b show the twins produced in the ferrite for specimens of types A and B, respectively, both subjected to drop heights of 5 m. Fig. 1b is a longitudinal section of the steel bar used and shows a banded structure of ferrite and pearlite signifying the possibility of some degree of rolling texture in the ferrite in all the specimens employed in this work.

It is reasonable to expect that for small extents of deformation the disposition of twins produced in the specimens will be symmetrical about the impact axis despite the fact that the steel possessed a banded structure and, therefore, likely some degree of rolling texture in the ferrite. This is because in each grain several twin systems are available so that the texture in the ferrite should not prevent twinning from occurring freely on the basis only of forces produced in the individual grains on the impact. The symmetrical disposition of twins about the impact axis has been experimentally verified with three further specimens, two of type A and one type B all impacted as before with drop heights of 5 m. These specimens were subsequently metallographically examined on sections made perpendicular to the impact direction and the angles ψ which twin traces made with one edge of the specimen were ascertained for 500 to 600 twin formations for each specimen. Actual number of twins considered were 558 and 520 for the two type A specimens and 545 for the single type B specimen. The percentage cumulative frequency of occurrence of the angles ψ were then plotted as in Fig. 2. The straight line in the figure shows the distribution in which various values of ψ occur with equal frequency, and refers to the case of a symmetrical distribution of twin orientations about the impact axis. It is seen that the experimental points for all the three specimens lie reasonably well on this line denoting that the twins in these specimens are more or less symmetrically distributed about the impact direction. Because of this it is possible to deduce the frequency distribution of the actual angles θ which the twins make with the impact direction from the experimental frequency distribution of apparent angles ψ for the various specimens sectioned parallel to the impact direction.

The method for doing this has been described by Scriven and Williams [3]. Scriven and Williams also made the point that planar features of finite dimensions have a lesser chance of being 'cut' by the metallographic plane (and therefore of being observed) when they are tilted towards greater parallelism with this plane. This "cutting probability" was shown to be given by $\cos\theta/\sin\psi$ for circular features which the deformation twins in the ferrite grains may be taken approximately to be. They were, however, considering a situation where such planar features were oriented as individuals independent of one another. In the present instance, the deformation twins generally occurred as formations of parallel twins considerably spaced apart within the ferrite grains so that the probability of their being intersected by the sectioning plane, and therefore of being detected, is, by and large, independent of their orientation. For this reason in this work the cutting probability has been taken to be 1 for all orientations of the twinning plane. Following this approach the cumulative frequency of

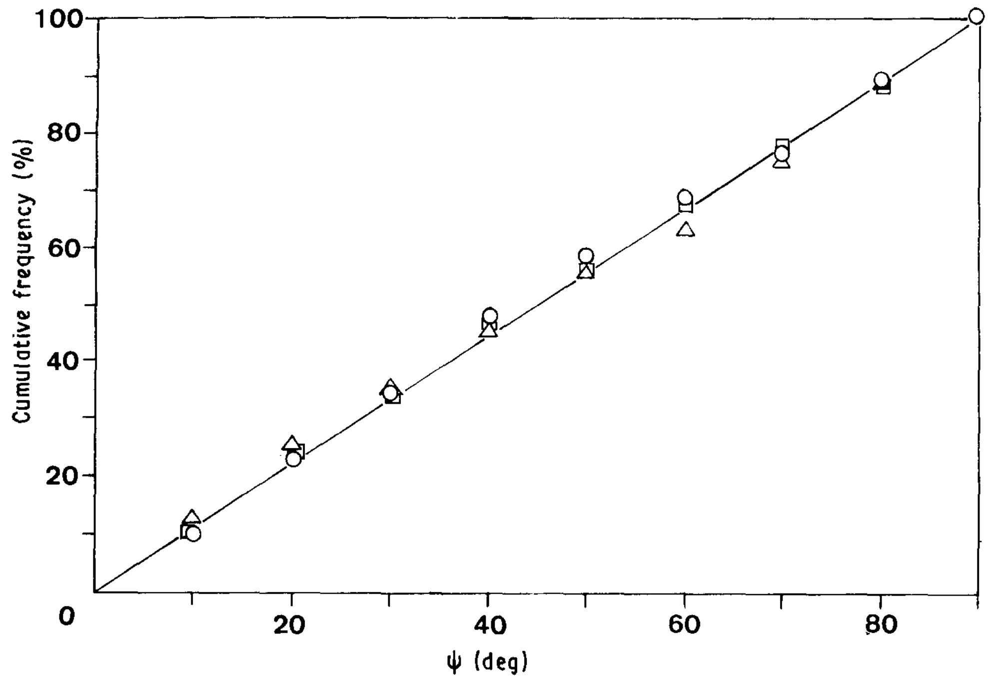

Figure 2 Percentage cumulative frequencies of the angle ψ made by twins with a fixed direction on a section perpendicular to the impact direction for drop heights of 5 m. The circular and square points are for two type A specimens and the triangular points are for a type B specimen.

occurrence of various θ values for the three type A specimens of drop height 4 m and the four type A specimens of drop height 5 m have been obtained and plotted in Fig. 3a and b, respectively. The method used to derive these θ frequency distributions was one equivalent to that of Scriven and Williams.

3. Discussion

The appearance of Fig. 3a and b indicate that there is a fair bit of scatter in the experimental distribution. This is perhaps to be expected because the polycrystallinity in the different specimens cannot be exactly identical and impact tests tend to have a certain degree of variability. When the average of the cumulative frequencies for each figure is taken, much smoother distributions are obtained as plotted in Fig. 4. The average is a population weighted average cumulative frequency of θ given by

$$\frac{\sum_{i=1}^{N} n_i f_i}{\sum_{i=1}^{N} n_i}$$

where f_i is the cumulative frequency of θ for the ith specimen and n_i the total number of twins considered for this specimen. N is the total number of specimens. The average distributions in Fig. 4 can be taken to represent some type of ideal behaviour about which actual behaviour is scattered around.

It is clear that the twins have not occurred freely in all orientations relative to the impact direction. When all orientations occur freely and randomly the cumulative frequency distributions of θ values would be given by

$$\int_0^\theta \sin\theta \, d\theta = 1 - \cos\theta$$

(see the analysis by Pellissier *et al.* [5]). The curve for this distribution has also been plotted in Fig. 4. The experimental distributions clearly do not conform to this freely and randomly occurring θ distribution.

Fig. 5 shows the averaged experimental θ frequency distribution for specimens type A, 5 m drop, as deduced from the experimental data in Fig. 4. The form of this distribution in which the frequency of θ is highest around the region of $\pi/4$ and falls to 0 for θ in the region of 0 and $\pi/2$ suggests that a minimum resolved shear stress requirement is operative. This is because the shear stress developing along twin planes of different orientations will be given by the Schmid equation

$$\tau = \sigma \cos\theta \cos\lambda \tag{1}$$

where τ is the shear stress, σ the applied compressive stress, and λ the angle between the twinning shear direction and the impact direction. From geometrical considerations λ will lie between $\pi/2 - \theta$ and $\pi/2$. For θ values in the region of 0 or $\pi/2$ the orientation or Schmid factor $\cos\theta\cos\lambda$ is, therefore, small and the resolved shear stress low. If the shear stress value is too low then twinning cannot be initiated and, thus, twinning will not occur, as observed, for θ values in the vicinity of 0 and $\pi/2$.

There appears to be conflicting views as to whether a unique critical shear stress exists for deformation twinning for a particular metal. Bell and Cahn [6], working with single crystals of zinc, and Priestner and Louat [7] and Priestner [8] working with large ferrite grains in a grain oriented 3.25% Si steel sheet, found that the shear stress required to initiate twinning depended on the orientation of the crystals or grains relative to the tensile axis. Hull [9], on the other hand, found a unique critical resolved shear stress to operate in twinning in ferrite crystals of 3.25% Si iron as did

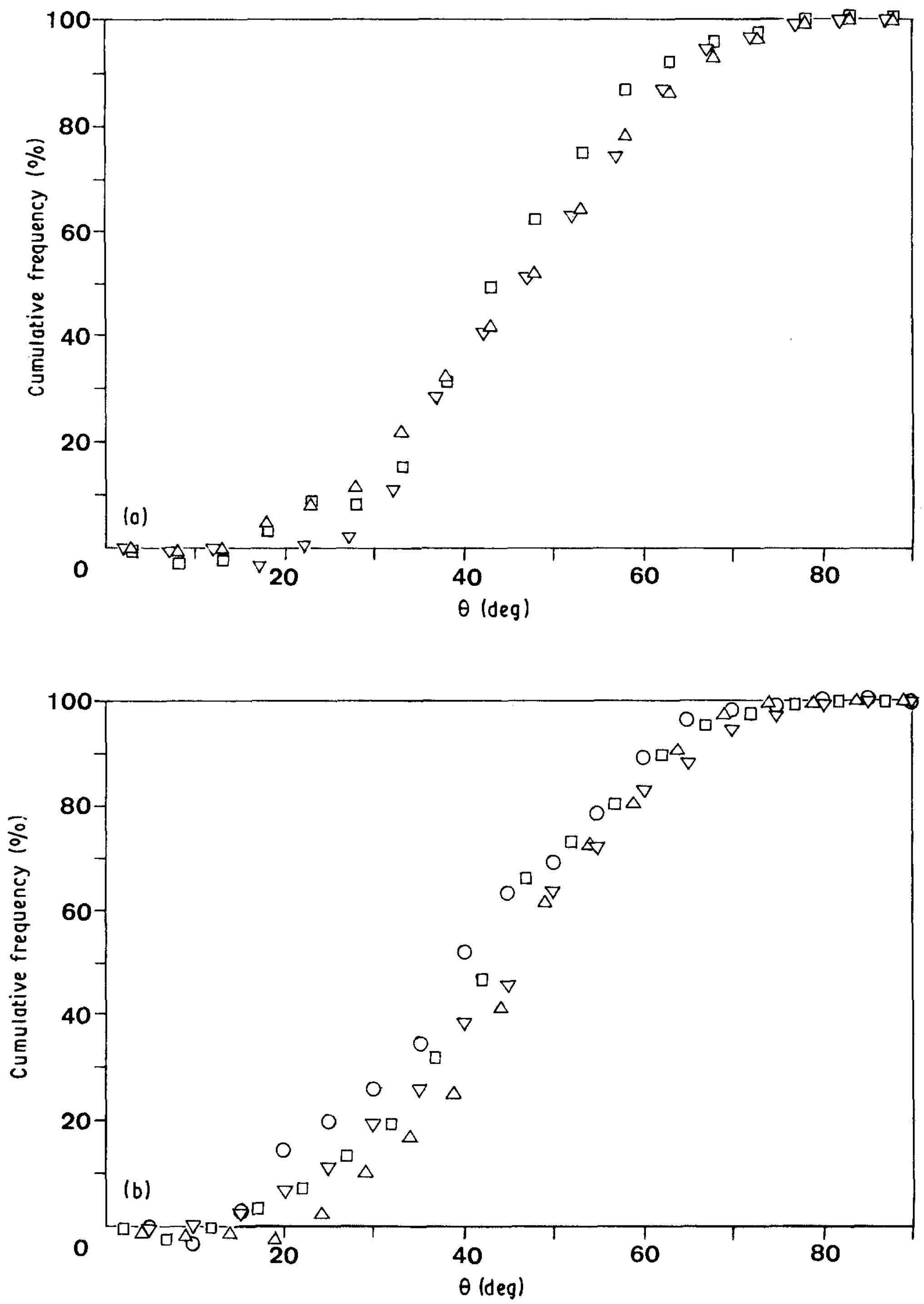

Figure 3 Percentage cumulative frequencies of θ, the angle of inclination of twin planes to the impact axis, for (a) three cases of type A specimens with drop heights of 4 m and (b) four cases of type A specimens with drop heights of 5 m. The different cases are distinguished by different types of points.

Bolling and Richman [10] for twinning in crystals of Fe–25 at.% Be and Fe_3Be. In this paper the frequency of occurrence of the angles θ will be predicted on the basis of a model which does not assume that a unique critical shear stress exists for twinning in ferrite at liquid nitrogen temperatures. Rather the assumption will be made that the Schmid factor $f = \cos\theta\cos\lambda$ must exceed some minimum value f_0 for twinning to be initiated. The basis for this assumption will now be explained.

If one refers to the work of Allen, Hopkins, and McLennan [11] on high purity iron f values for 21 twins were reported and the lowest value was 0.21. In the work of Bolling and Richman [10] on Fe–25 at.% Be specimens on 47 twin cases the lowest f value was 0.29. In the work of Bell and Cahn [6] on pure zinc the lowest f value was 0.27 for 17 single crystal specimens which twinned. If one considered the graphic results of Reed-Hill [12] where a statistical study of deformation twins in polycrystalline zirconium was made again one would conclude that the f value below which twinning was not seen was a value somewhere in the range 0.2 to 0.3.

Thus for a number of quite different materials results of some previous work over considerable numbers of twin cases investigated have shown that the value of f associated with twinning does not fall below a value somewhere in the range of 0.2 to 0.3. That different

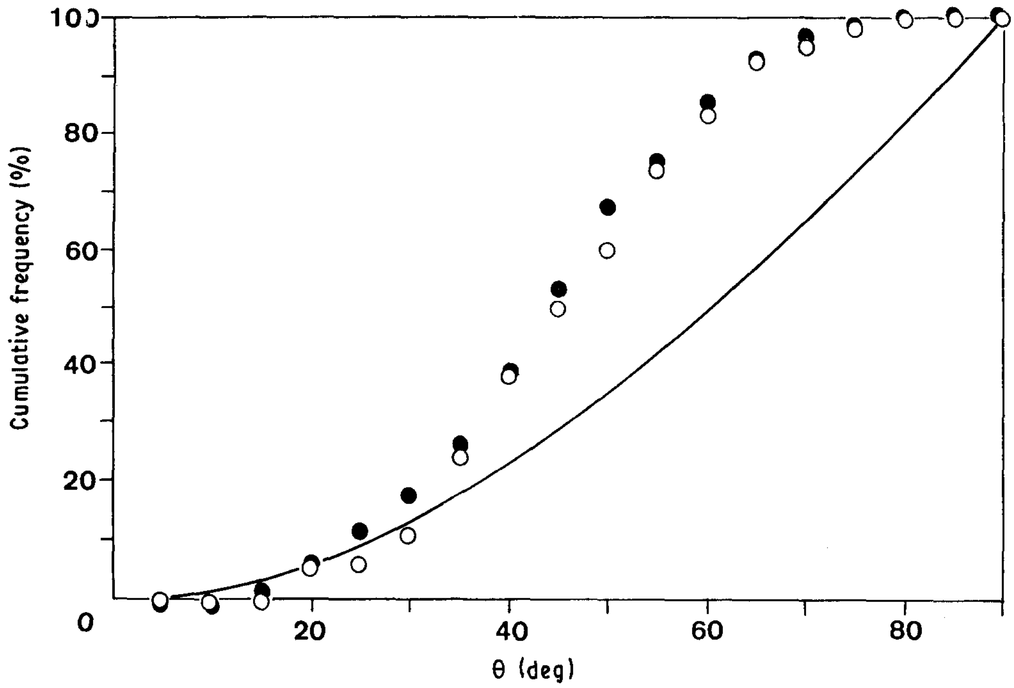

Figure 4 The population weighted average percentage cumulative frequencies of θ for the three cases in Fig. 3a and for the four cases in Fig. 3b. The curve gives the cumulative frequency distribution for the case where the twin plane orientations are perfectly random (○ 4 m drop, ● 5 m drop).

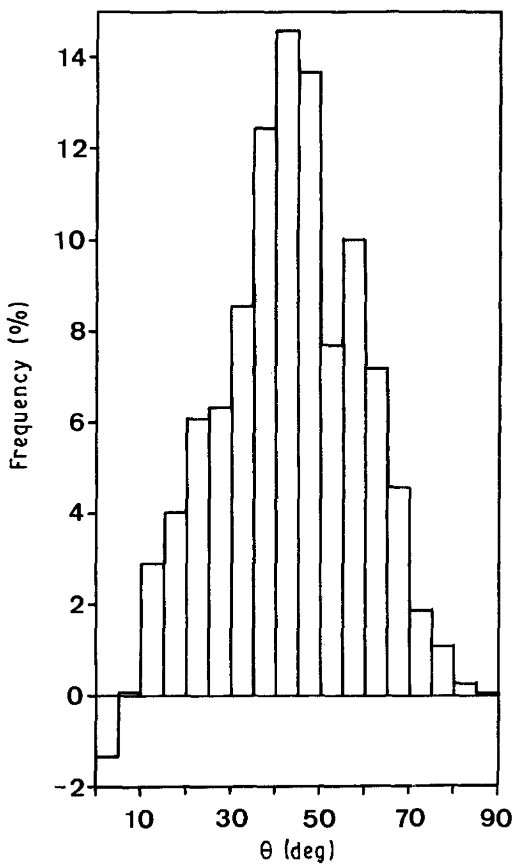

Figure 5 θ percentage frequency plot corresponding to the cumulative frequency of θ average plot for the 5 m drop in Fig. 4.

materials should exhibit about the same minimum f value of somewhere between 0.2 and 0.3 for twinning is perhaps not surprising. The range of operative applied stress values is from the yield stress value y_s to the ultimate stress y_u. y_u is usually not more than about $2y_s$. At the yield stress value mainly twins on planes with the most favourable Schmid factors ($\cong 0.5$) will be initiated. If the applied stress σ is increased to the ultimate stress y_u then twins on planes with f values down to 0.25 but not below this may now be initiated since

$$f_u/f_s = y_s/y_u = 0.5 \tag{2}$$

where f_u and f_s are the minimum f values corresponding to applied stresses of y_u and y_s, respectively, thus, irrespective of the material, f should not fall below about 0.25 for twinning.

On the basis that a minimum Schmid factor value, f_0, exists for twinning in ferrite, and on the reasonable assumption that $\{1\,1\,2\}$ ferrite planes are found in equal numbers in any orientation in the specimens being investigated, it is possible to develop an analysis to predict the relative frequency of occurrence of different values of the angle θ between twin plane normals and the impact direction. This analysis is presented in Appendix I. The result of the analysis is that the probability of occurrence of twinning on $\{1\,1\,2\}$ planes with normals lying at angles in the range θ_1 to θ_2 with the impact direction is

$$P(\theta_1, \theta_2) = \frac{\int_{\theta_1}^{\theta_2} \cos^{-1}(2f_0/\sin 2\theta)\sin\theta\, d\theta}{\int_{\theta_0}^{\pi/2-\theta_0} \cos^{-1}(2f_0/\sin 2\theta)\sin\theta\, d\theta} \tag{3}$$

where $\theta_0 = \frac{1}{2}\sin(2f_0)$, and θ_1 and θ_2 should lie between θ_0 and $\pi/2 - \theta_0$, since no twinning can occur for $\theta < \theta_0$ and $\theta > \pi/2 - \theta_0$.

The cumulative frequency distribution of θ given by Equation (3) has been plotted for various values of f_0 in Fig. 6 together with the experimental data of Fig. 4. Again it is seen that although these curves with an appropriate value of f_0 can be much closer to the

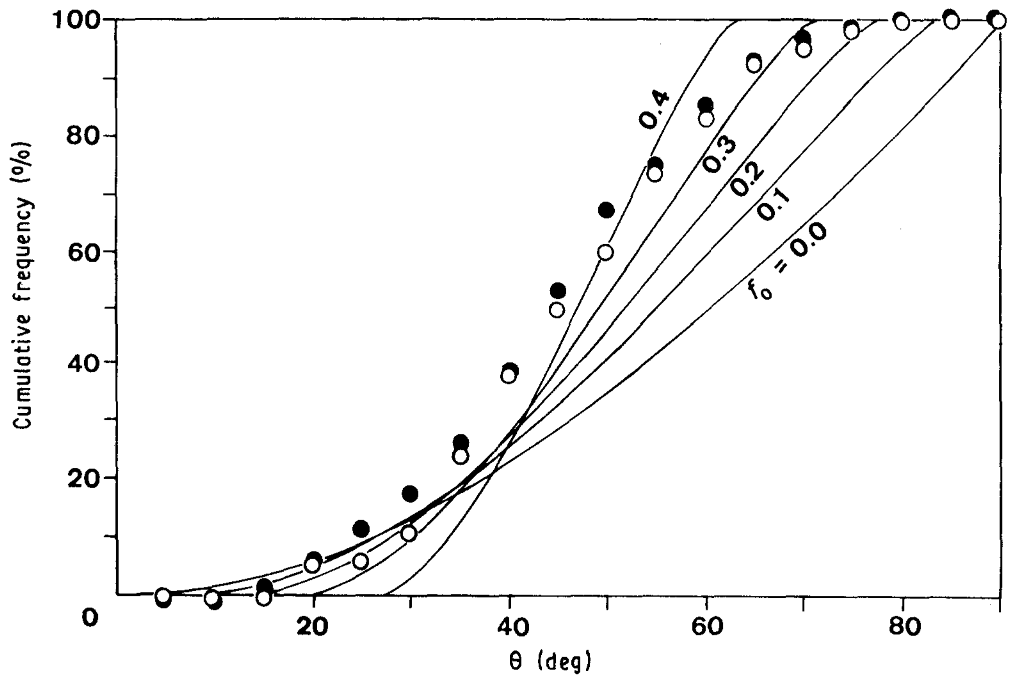

Figure 6 θ cumulative frequency curves from Equation 3 of text for various minimum Schmid factor values, f_0, plotted alongside the experimental cumulative frequency data points of Fig. 4. The f_0 value of 0.0 corresponds to the randomly occurring θ curve of Fig. 4 (○ 4 m drop, ● 5 m drop).

observed average experimental distribution than for the case of a freely occurring θ distribution, for no value of f_0 do the curves match the experimental distribution well.

It is reasonable to assume that twinning with shear opposing the applied compressive force is not favoured. This indicates that the direction of the applied force itself influences the orientation of planes along which twinning may occur. A little consideration will show that the greater tan θ is compared to 1 the larger will be the twin shear component along the impact direction as compared to the other component resolved at right angles to the impact direction, and therefore, we may expect a greater resistance of the applied compressive force direction to twinning. Tan θ, in fact, is the ratio of the size of the twin shear component along the impact direction to that perpendicular to the impact direction. We shall make the assumption that when $\tan\theta > 1$ twinning occurs with greater difficulty and the degree of twinning is reduced by the factor $1/\tan\theta$, i.e., the greater tan θ is compared to 1 the greater in proportion is the twinning difficulty.

Mathematically, the probability of occurrence of twins whose normals lie at angles between θ_1 and θ_2 to the impact direction as given by Equation 3 can now be expressed as

$$P(\theta_1, \theta_2) = \frac{\int_{\theta_1}^{\theta_2} R\cos^{-1}(2f_0/\sin 2\theta)\sin\theta\,d\theta}{\int_{\theta_0}^{\pi/2-\theta_0} R\cos^{-1}(2f_0/\sin 2\theta)\sin\theta\,d\theta} \quad (4)$$

where

$$R \begin{cases} = 1/\tan\theta & \text{for } \theta > \pi/4 \\ = 1 & \text{for } \theta \leqslant \pi/4. \end{cases}$$

The cumulative distribution curves given by this equation have been plotted out in Fig. 7 alongside the experimental average distributions of Fig. 4. In the case of the 4 m drop specimens an f_0 value of 0.25 was used giving the best fitting curve. In the case of the 5 m drop specimens the best fit is given by an f_0 value of 0.22. It is seen that the fits are very good. The f_0 values for the two drop heights are about the same and suggests that the experimental distributions too are not much different from each other. This is not surprising as the impact velocities for the drop heights of 4 and 5 m are 8.9 and 9.9 m s^{-1}, respectively. These velocities differ by only a small amount and the effect this difference produces is expected to be small and to be within the statistical scatter of the experimental distributions.

The stress on impact is given by Johnson [13] as

$$\sigma = \frac{Y + \rho c v}{(1 + m/M)ct/L} \quad (5)$$

where m/M is the ratio of specimen mass to the drop weight mass, Y the yield strength of the steel, ρ density of the steel, c the plastic wave velocity, and v the velocity of the weight just before impact. L is the length of the specimen in the direction of impact and t the time after impact. Because in the present work m/M is 0.008, a very small quantity, and the specimen is short, there will be several reflections of the propagating stress giving a maximum stress in the specimen which may be taken to be

$$\sigma_{max} = Y + n\rho c v \quad (6)$$

where n is a factor (probably several times greater than 1) to account for the enhancement of stress by the reflections at the ends of the specimen. Plastic deformation will cease when the yield strength of the specimen builds up by strain-hardening to a level greater than the propagating stress. Since the plastic strain found experimentally for the drop tests was

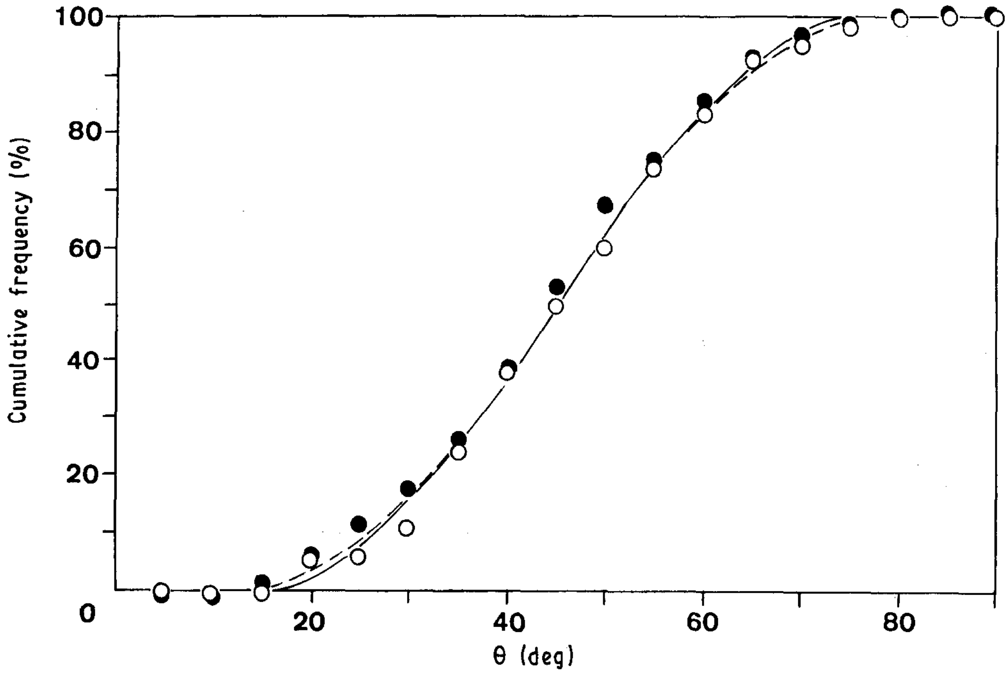

Figure 7 θ cumulative frequency curves with twin suppression factor, $1/\tan\theta$, incorporated according to Equation 4 with $f_0 = 0.22$ (broken curve) and 0.25 (full curve) plotted over the experimental data points of Fig. 4 (○ 4 m drop, ● 5 m drop).

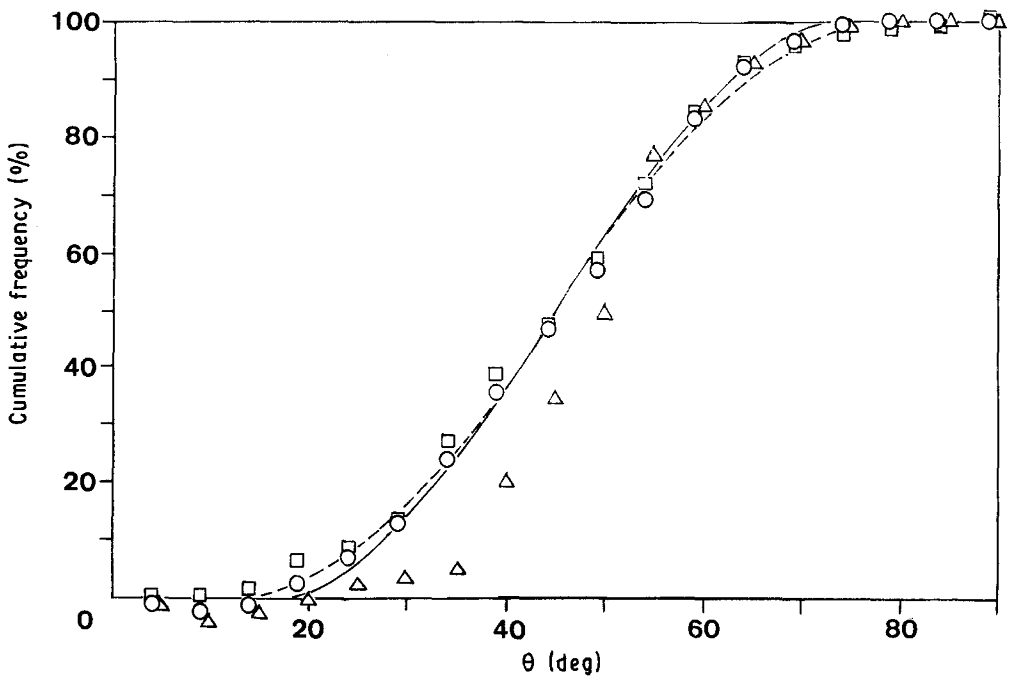

Figure 8 The θ cumulative frequency curves from Equation 4 with $f_0 = 0.22$ (- - - - -) and 0.28 (——). Experimental values for type A, 3 m drop (□), and type B, 4 m drop (Δ) and 5 m drop (○) have also been plotted.

small the term $n\rho cv$ should be small or, at best, comparable to Y in size. For drops of 3 and 5 m the velocities on impact of the drop weight are 7.7 and 9.9 m s^{-1}. Thus, assuming the factor n in Equation 6 will not change significantly with such close velocities, the ratio of maximum stress for a 5 m drop to that for a 3 m drop would be expected to be greater than 1 but somewhat smaller than $9.9/7.7 = 1.3$. If the f_0 value for a drop of 5 m is 0.22 then that for a drop of 3 m would be more than 0.22 but certainly not more than $0.22 \times 1.3 = 0.28$. Note that

$$f_{03}/f_{05} \;=\; \sigma_5/\sigma_3 \qquad (7)$$

where σ_3 and σ_5 are the maximum stresses developed for drops of 3 and 5 m, respectively, and f_{03} and f_{05} are the corresponding minimum possible f values for twinning.

In other words f_0 is not expected to change very much for the different drop heights used in this work. The difference in the cumulative θ frequency distribution curves given by Equation 4 for $f_0 = 0.22$ and 0.28 is shown in Fig. 8. It is seen that even if f_0 did

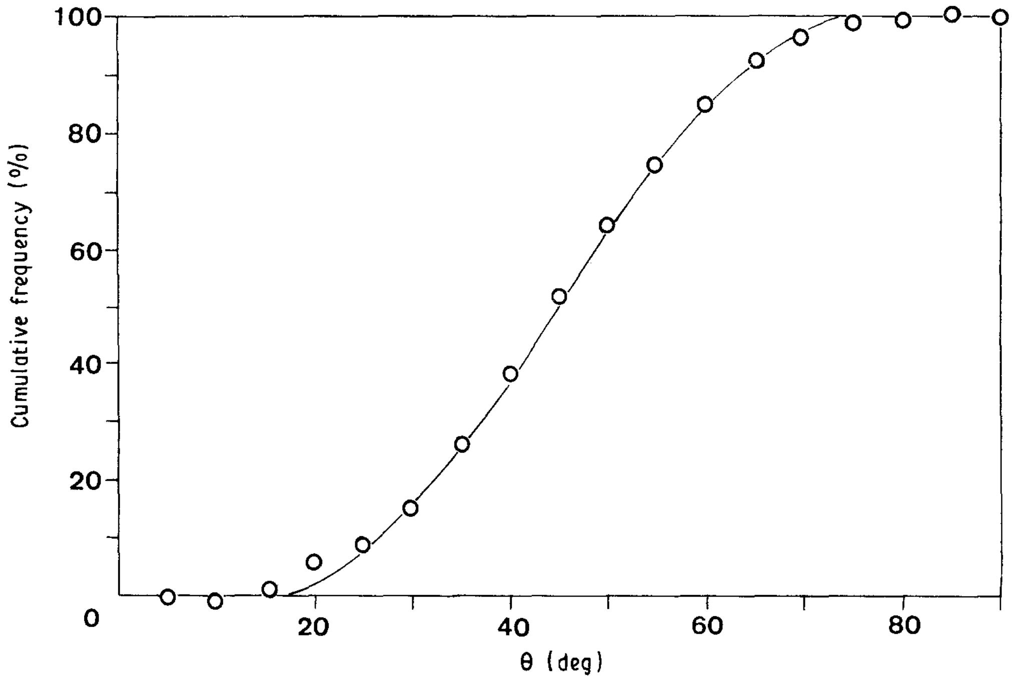

Figure 9 Circular points are the population weighted average cumulative frequencies of all the type A specimens in Fig. 4, 4 m and 5 m drops, as well as the one specimen for a 3 m drop height. The curve is that computed from Equation 4 with $f_0 = 0.25$.

change from 0.22 to 0.28 the distributions do not change significantly. Thus one may view all the specimens of type A for the different drop heights of 3, 4, and 5 m to have more or less the same distribution.

This is borne out experimentally when the population weighted average of all the specimens of type A for the 3, 4, and 5 m drops are taken together. As seen in Fig. 9a smoother distribution is obtained and the curve according to Equation 4 with $f_0 = 0.25$ passes through these points extremely well.

Fig. 8 also shows the experimental θ frequency data for the specimen of type A dropped 3 m and specimens type B dropped 4 and 5 m. The type A, 3 m drop, and type B, 5 m drop, data fit well with the curve of the type given by Equation 4 with f_0 in the region of 0.25 but the type B, 4 m drop, data seems to suffer a scatter as for some of the earlier seen specimens (Fig. 3a and b). It would appear that the θ frequency distribution behaviour for type B specimens is the same as for the type A specimens so that the direction of the rolling axis relative to the impact direction has no great effect on the nature of the θ frequency distribution. This is to be expected because of the considerable number of $\{1\,1\,2\}$ planes available for twinning in each ferrite grain.

4. Conclusion

The nature of initial deformation by twinning of a mild steel specimen, under a unidirectional compressive force, is one of deformation proceeding with equal likelihood along $\{1\,1\,2\}$ planes in any orientation except those for which the Schmid factor $\cos\theta \cos\lambda$ is less than about 0.25 (below which twinning cannot be initiated) and with some suppression for twin plane orientations where the twin shear opposes the applied compressive force. The suppression consists of a diminishing of the probability of occurrence of twins on these planes by a factor of $1/\tan\theta$ where θ is the angle between the twin plane normal and the compressive force direction. Hence, local deformations which causes shear or strain opposing the shaping action of the applied force are disfavoured. The greater the opposition, the greater the disfavour. The concept of a threshold value of the Schmid factor for twinning holds well with the experimental data and the threshold value of about 0.25 is consistent with the experimental Schmid data of other work such as those of Allen *et al.* [11], Bolling *et al.* [10], Bell *et al.* [6], and Reed-Hill [12]. Although the results here pertain to small deformations, they may also be taken to represent the initial phase of more severe deformation treatments.

Appendix 1

In Fig. 10 $OXYZ$ is a rectangular co-ordinate system with plane OYZ parallel to a vertical face of the impacted specimens and OZ parallel to the direction of impact of the falling weight. ABCD represents a $\{1\,1\,2\}$ plane in a ferrite grain in the specimen with normal ON inclined at an angle θ to OZ. The projection OM of ON onto the OXY plane makes an angle ϕ with OX. OE is a direction in ABCD coplanar with OZ and ON. OS, the shear direction for twinning on ABCD makes an angle λ with OZ and α with OE. OT is the intersection of ABCD with the plane OYZ and therefore gives the trace direction of a deformation twin on ABCD as seen on a vertical section of the specimen. The angle ψ between this trace and the impact axis OZ is also shown in Fig. 10.

Let σ be the maximum stress developed during impact. If plastic strain is small it may be assumed that grain accommodation effects are negligible. The shear stress along ABCD in the direction of the twinning shear OS is then $\sigma\cos\theta\cos\lambda$. For twinning to occur

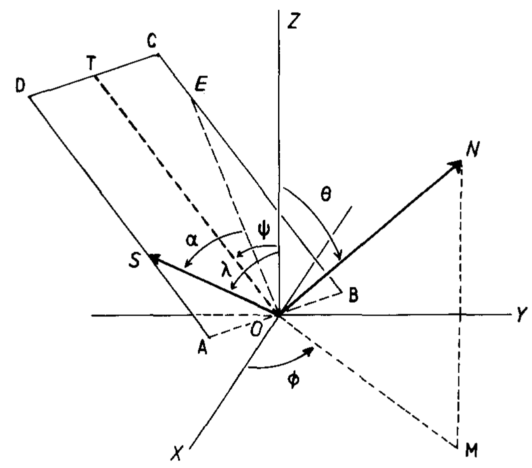

Figure 10 Schematic diagram of twin plane and twin shear direction in a ferrite grain and their geometrical relationships with the impact direction and specimen surface of examination.

along ABCD

$$\cos\theta\cos\lambda > f_0 \quad \text{(A1)}$$

where f_0 is the minimum value of $\cos\theta\cos\lambda$ for twinning to occur under the applied stress σ.

For a given value of θ, λ can have values between $\pi/2-\theta$ and $\pi/2$ only. The maximum value of the Schmid factor $\cos\theta\cos\lambda$ occurs when $\lambda=\pi/2-\theta$. Thus if $\cos\theta\cos(\pi/2-\theta)<f_0$ twinning will not occur. Otherwise, twinning may occur for values of λ from $\pi/2-\theta$ to some value λ' where

$$\cos\theta\cos\lambda' = f_0 \quad \text{(A2)}$$

As λ varies from $\pi/2-\theta$ to λ', the angle α varies from 0 to α' (say). It is shown in Appendix 2 that

$$\alpha = \cos^{-1}(\cos\lambda/\sin\theta) \quad \text{(A3)}$$

so that

$$\begin{aligned}\alpha' &= \cos^{-1}(\cos\lambda'/\sin\theta)\\ &= \cos^{-1}(f_0/\sin\theta\cos\theta)\\ &= \cos^{-1}(2f_0/\sin 2\theta) \quad \text{(A4)}\end{aligned}$$

For $\sin 2\theta < 2f_0$ no value of λ exists for which the condition in Equation A1 necessary for twinning is possible and therefore the range of θ for which twinning will occur is $\theta_0<\theta<\pi/2-\theta_0$ where

$$\theta_0 = \sin^{-1}(2f_0)/2 \quad \text{(A5)}$$

Even in an approximately random collection of ferrite grains, because of the multiplicity of $\{1\,1\,2\}$ planes, there will be an equal likelihood of finding $\{1\,1\,2\}$ planes with different values of α. The probability of finding twins on $\{1\,1\,2\}$ planes with normals at an angle θ to the impact axis is, therefore, the probability that the twin shear direction OS will be at angle α to OE which is between 0 and α'. This probability will be proportional to α'.

The number of $\{1\,1\,2\}$ planes among the many ferrite grains having normals inclined at an angle between θ and $\theta+d\theta$ with the impact direction is, on reference to Pellissier *et al.* [5], proportional to $\sin\theta\,d\theta$.

Noting Equation A4, the number of these $\{1\,1\,2\}$ planes in which the resolved shear stress along the twin shear direction will be sufficient for twinning will be proportional to

$$\cos^{-1}(2f_0/\sin 2\theta)\sin\theta\,d\theta$$

It follows that the probability of finding twins with twin plane normals at angles between θ_1 and θ_2 with the impact axis is as given in Equation 3, where θ_1 and θ_2 have values between θ_0 and $\pi/2-\theta_0$.

Appendix 2

In Fig. 11 are three non-coplanar directions OZ, OE, and OS making angles of α, β, and λ with one another as shown. There is a fourth direction ON, perpendicular to both OE and OS, and at an angle of θ with OZ. For this geometry the following relationship has been established [14]

$$\cos^2\theta = 1-(\cos^2\lambda+\cos^2\beta - 2\cos\lambda\cos\beta\cos\alpha)/\sin^2\alpha \quad \text{(A6)}$$

This will give

$$\sin^2\alpha\sin^2\theta = \cos^2\lambda+\cos^2\beta-2\cos\lambda\cos\beta\cos\alpha \quad \text{(A7)}$$

If OZ should lie in the plane NOE, then $\beta=\pi/2-\theta$, and the directions OE, OS, OZ, and ON become identical with the same named directions in Fig. 10. In this case Equation A7 becomes

$$\sin^2\alpha\sin^2\theta = \cos^2\lambda+\sin^2\theta-2\cos\lambda\cos\theta\cos\alpha \quad \text{(A8)}$$

$$\sin^2\theta\cos^2\alpha - 2\cos\lambda\sin\theta\cos\alpha+\cos^2\lambda = 0 \quad \text{(A9)}$$

$$(\sin\theta\cos\alpha-\cos\lambda)^2 = 0$$

$$\cos\alpha = \cos\lambda/\sin\theta.$$

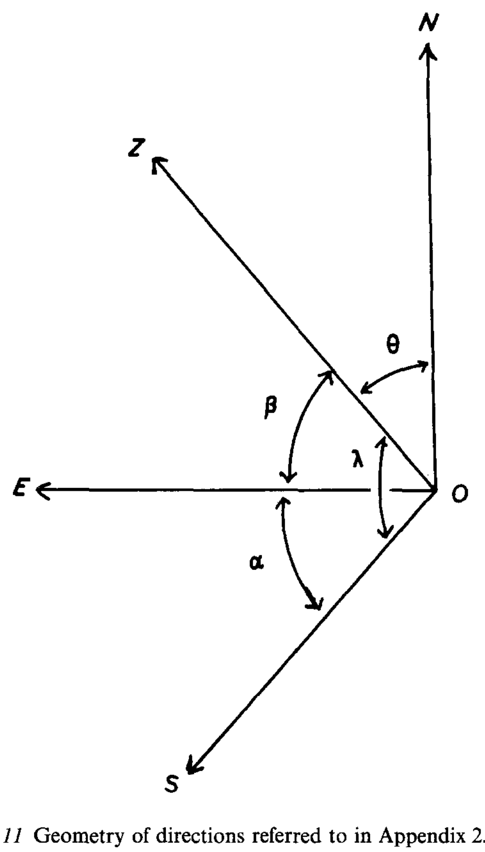

Figure 11 Geometry of directions referred to in Appendix 2.

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